TRINITY COLLEGE

Year 12 Mathematics Specialist 3/4
Test 4 2022

Weighting: 7%

Scientific Calculator Assumed
Integration

STUDENT'S NAME ____

DATE: Monday 25 July **TIME:** 50 minutes **MARKS**: 50

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser
Special Items: Three **Scientific** calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

---

1. (6 marks)

Using the given substitutions or otherwise, determine the following integrals

(a) $\int (\cos x)\cos(2\sin x)\,dx$ let $y = 2\sin x$ [3]

(b) $\int \dfrac{\sin 2x}{4 + 3\cos^2 x}\,dx$ let $y = 4 + 3\cos^2 x$ [3]

2. (11 marks)

(a) Determine $\int \frac{3\cos x - 4\sin x}{3\sin x + 4\cos x}\,dx$ using the integral: $\int \frac{f'(x)}{f(x)}\,dx = \ln\ |f(x)| + c$ [1]

(b) For the equation $\sin x = a(3\sin x + 4\cos x) + b(3\cos x - 4\sin x)$.

(i) two simultaneous equations can be formed. Given that one of the equations is : $3a - 4b = 1$. Determine a second equation and solve for *a* and *b*. [3]

(ii) hence, using (b)(i), show that: [3]

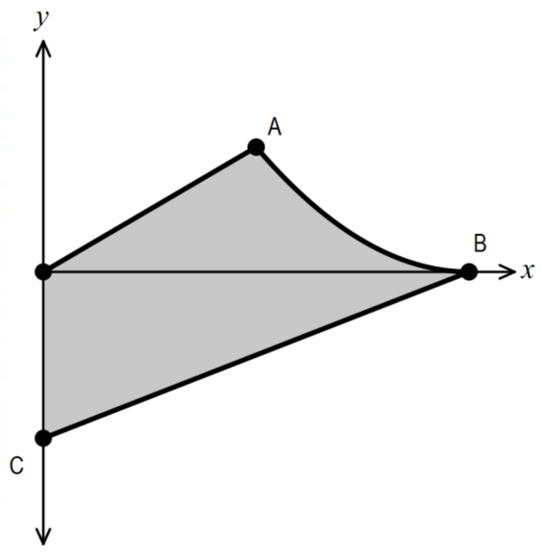
$$\frac{\sin x}{3\sin x + 4\cos x} = \frac{3}{25}\left(1 - \frac{4}{3}\left(\frac{3\cos x - 4\sin x}{3\sin x + 4\cos x}\right)\right)$$

(c) Hence, show $\int_0^{\frac{\pi}{2}} \frac{\sin x}{3\sin x + 4\cos x}\, dx = \frac{1}{50}\left(3\pi + 8 \ln\left(\frac{4}{3}\right)\right)$ [4]

3. (8 marks)

Consider the following graphs of the functions $y = \frac{3x}{2}$, $y = \frac{3}{2}(x-2)^2$ and $y = x - 2$.

Determine



(a) The coordinates of the points A, B and C. [3]

(b) An integral, or a combination of integrals that will give the area of the shaded region. (Do not evaluate) [2]

(c) The exact area of the shaded region. [3]

4. (6 marks)

Using the substitution $x = \sin^2 \theta$, show that $\int_0^{\frac{1}{2}} \sqrt{\frac{x}{1-x}}\ dx = \frac{\pi}{4} - \frac{1}{2}$

5. (12 marks)

(a) Determine the exact area of the finite region enclosed by the line $y = 4x$ and the curve $y^2 = 16x$. [4]

(b) Determine the exact volume generated when this region is rotated completely about

(i) the x-axis [4]

(ii) the y-axis [4]

6. (7 marks)

(a) Using the substitution $u = \sin x$, show that $\displaystyle\int \frac{\cos x}{3 + \cos^2 x}\, dx = \int \frac{du}{4 - u^2}$ [3]

(b) Hence, using partial fractions, show that $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\cos x}{3 + \cos^2 x}\, dx = \frac{1}{4} \ln 3$ [4]